Kalman-Filter Channel Estimator for OFDM System In Time-Varying Channel

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***Abstract*:** This paper presents a method of improving the least square channel estimator accuracy without increasing the pilot density. The Kalman filter process equation can be represented as Autoregressive model, and the least square channel estimate is seen as a noisy measurement of the true channel state, so the Kalman filter measurement equation can be represented as the least square estimated channel. The process equation and the measurement equation jointly form a state space model of the dynamic of the channel. Thus the Kalman filter can be used to estimate the state variable, i.e., the time varying channel. The Kalman filter based channel estimator leads to a significant gain in performance as compared to the least square channel estimator.

Key words: *least square channel estimator, OFDM system, Kalman filter.*

I- Introduction

Kalman filter has been applied to estimate the channel of OFDM systems in the frequency domain dimension in [4].Although the proposed algorithm utilizes time and frequency domain channel correlation at the same time, and achieves optimum performance., Its drawback is the high complexity due to the high dimension of the state variable, which can be significantly high when there are large number of subcarriers. One solution to reduce the complexity of the Kalman filter channel estimator in [4] is to implement it at a per- subcarrier fashion [5,6]. In [6] a per-subcarrier Kalman filter based on comb-type pilot arrangement is applied to estimate of fast and frequency selective fading channel. However, the per-subcarrier Kalman filter only uses the time-domain correlation of the channel fading and fails to take advantage of the frequency-domain correlation. Therefore, the proposed algorithms in [5,6] refine the "rough" channel estimation in per-subcarrier through exploiting the frequency domain channel correlation. The received pilot signals is used directly in Kalman filter algorithm as the kalman measurement equation in [4,5], which increases the computational complexity of theses algorithms, since the pilot signal recursively will be used in these algorithms.

II Channel estimation algorithm

Given the state space model for the time varying channel to be estimated as follows:

$The\ process\ model$

$$H^p(n) = A\, H^p(n-1) + V^p(n)$$

$The\ measurement\ model$

$$H^p_{LS}(n) = H^p(n) + \ {W^p}_{LS}(n)$$

The Kalman filter algorithm can be applied to solve this state-space model to obtain the estimate of the channel gain $H^p(n)$. The channel estimation algorithm based on Kalman filter can be described in four steps LS channel estimation, filter initialization, time update, and measurement update.

Step 1: LS channel estimation

$$H^p(n)_{LS} = \frac{Y^p(n)}{X^p(n)} \quad \text{(LS channel estimate)}$$

Step 2: Filter initialization

$$\widehat{H}^p(0) = 0_p \quad \& \quad \hat{p}(0) = I_p \qquad \text{(Initial conditions)}$$

Where 0_p and I_p are $N_p \times N_p$ null and identity matrixes respectively

Step 3: Time update

$$\widetilde{H}^p(n+1) = \ A\, \widehat{H}^p(n) \quad \text{(State prediction)}$$

$$\tilde{p}(n+1) = A\hat{p}(n)A^H + Q \quad \text{(Prediction error covariance)}$$

Step 4: Measurement update

$$K = \tilde{p}(n)(\tilde{p}(n) + R)^{-1} \quad \text{(Kalman Gain)}$$

$$\widehat{H}^p(n) = \ \widetilde{H}^p(n) + K\big(\ H^p_{LS}(n) - \widetilde{H}^p(n)\,\big) \quad \text{(State estimation)}$$

$$\hat{p}(n) = \tilde{p}(n)(I - K) \qquad \text{(Estimation error covariance)}$$

III Description of simulation

OFDM system parameters used in the simulation are shown in Table 1.

Table 1. OFDM system parameters

Parameters	**Specification**
FFT size(N_{FFT})	256
Nominal Channel Bandwidth, $Bw_{nominal}$	5 MHz
Number of data Subcarriers, N_{data}	174
Number of pilot Subcarriers, N_{pilot}	26
Number of guard Subcarriers, N_{guard}	56
Sampling Factor, n	144/125
Guard Interval, G	1/4 from symbol period
Number of used Subcarriers , N_{used}	$N_{data} + N_{pilot} = 200$
Sampling Frequency, f_s	5.76 MHz
Subcarrier Spacing, Δf	$\Delta f = \frac{f_s}{N_{FFT}} = 22.5\ KHz$
Used bandwidth, Bw_{used}	$Bw_{used} = N_{used} \times \Delta f = 4.5$ MHz
Useful Symbol Time, T_s	$T_s = \frac{1}{\Delta f} = 44.444$ µs
Cyclic Prefix Time, T_{CP}	$T_{CP} = G \times T_s = 11.111$ µs
OFDM Symbol Time, T_{sym}	$T_{sym} = T_s + T_{CP} = 100$ µs
Signal Constellation	QAM &PSK
Pilot Ratio	1/8
Channel model	Rayleigh (Jakes spectrum)
Number of Channel taps (L)	3
Maximum delay spread	35 µs

IV. Channel Model Used in Simulations

The wireless channel is assumed to be a multipath Rayleigh fading channel corrupted by additive white Gaussian noise (AWGN), consisting of L paths

$$h(\tau) = \sum_{l=0}^{L-1} \alpha_l \delta(t - \tau_l)$$

Where α_l is Rayleigh distributed channel taps, each path of the channel is modeled with a Jakes Doppler spectrum with a maximum Doppler shift of 70 Hz. In this thesis, the discrete-time channel impulse response is assumed to have 3 paths, in which the first fading path always has a zero-delay, and other fading paths have delays that are always less than the length of cyclic prefix CP. Table 4.2 gives paths delays and power of channel used in Simulations.

Table 2. Path delays and power for channel Used in Simulations

Path	**Delay (µs)**	**Power (dB)**
1	0	0
2	0.1	-5
3	0.3	-10

V. Simulation Results

The performances of the Least square and Kalman filter channel estimators are compared in figure 1 .The result shows that the Kalman filter algorithm achieved about 3-4 dB signal to noise ratio improvement compared to the LS method. It is worthy *of* mention, that improvement in term signal to noise ratio obtained without increasing the pilot density, as a result there is no more waste of the used bandwidth.

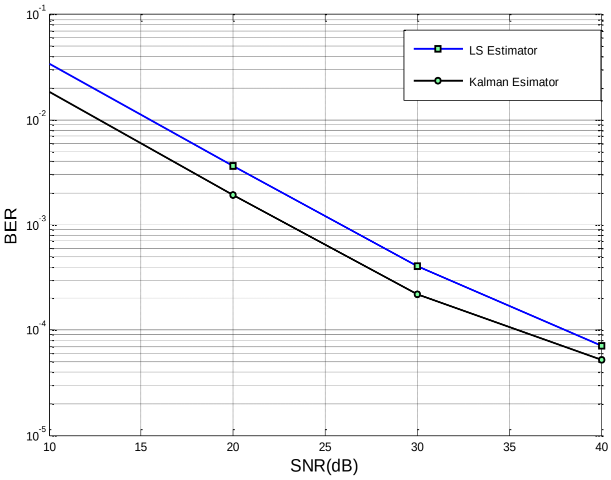

Figure 1. Performance of Least Square vs. Kalman channel estimators

Applying the described OFDM system with different Doppler frequencies, it can be noticed that the performance is worse when the Doppler frequency is increased for Least square as shown in figure 2, either for Kalmen estimator as shown in figure 3.

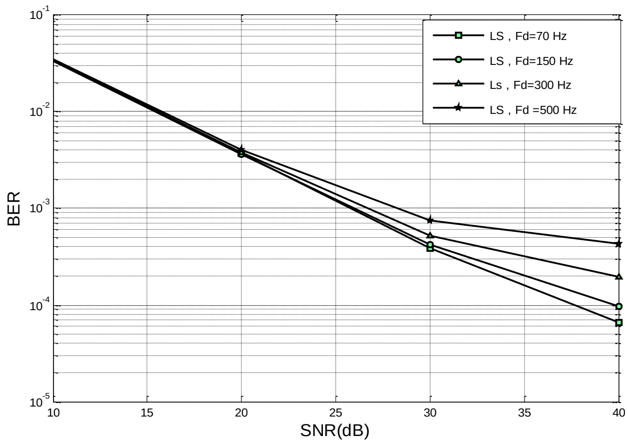

Figure 2. Doppler frequency effect on the least square estimator

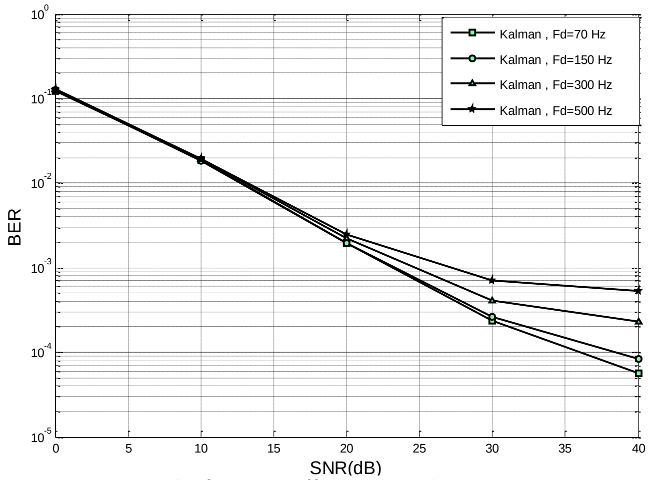

Figure 3. Doppler frequency effect on the Kalman estimator

Figure 4 and 5 show the performance of the different interpolation methods with Least Square and Kalman channel estimations respectively. The performance of interpolation techniques ranges from the best to the worst method as follows: spline, linear, nearest neighbor.

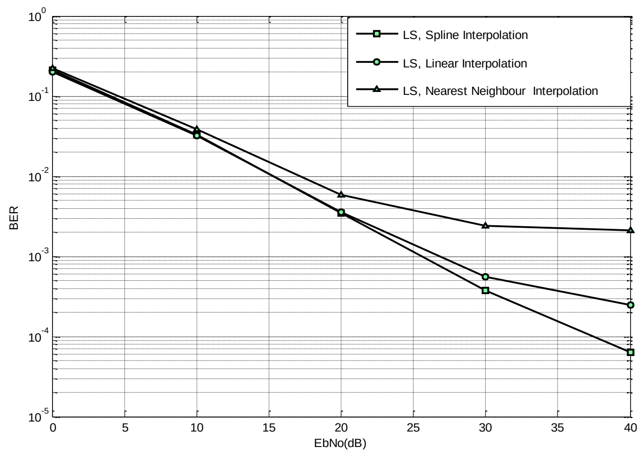

Figure 4. Interpolation methods using Least Square estimator

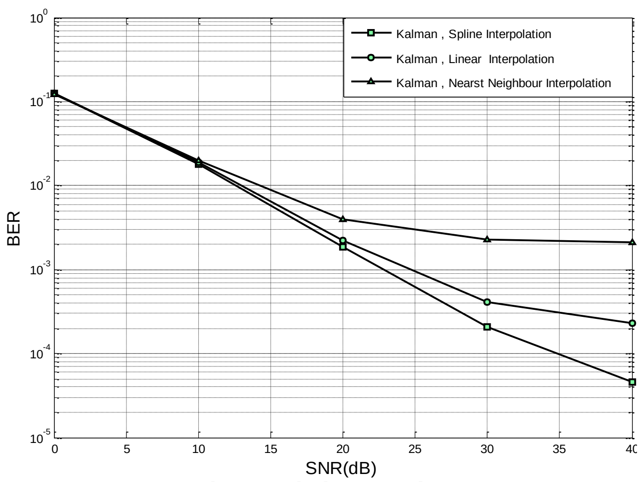

Figure 5. Interpolation methods using Kalman estimator

Figure 6 and 7 show the performance of the simulated system using Least Square and Kalman estimator methods with MPSK and MQAM for modulation order of 2,4,16, and 64.

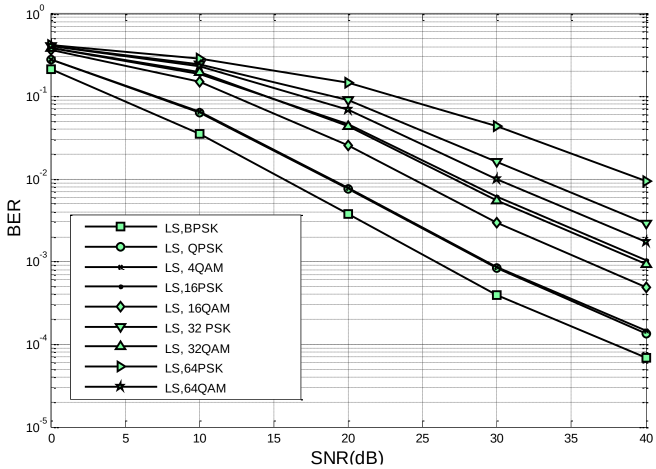

Figure 6. Performance of MPSK and MQAM modulation schemes, least square estimator

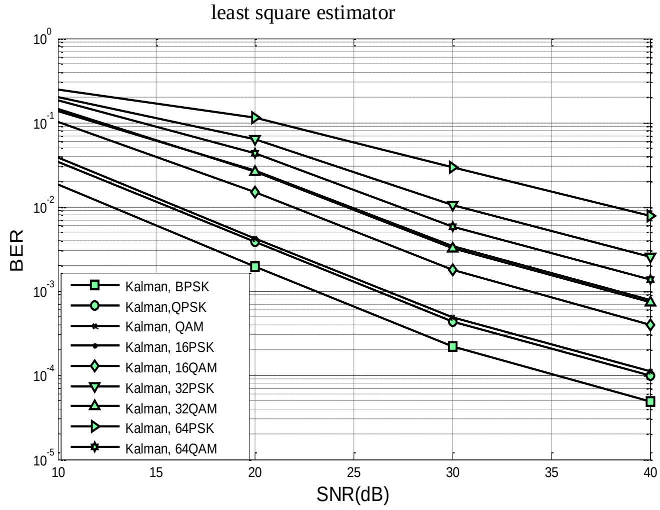

Figure 7. Performance of MPSK and MQAM modulation shcems ,Kalman estimator

VI. Conclusion

The main conclusion of this paper is the improvement of least square channel estimator without increasing the pilot density. The channel frequency response on the pilot subcarriers is obtained using least square channel, and then the estimated channel frequency response is improved by Kalman filter algorithm; and interpolated over frequency coefficients to the data symbols. The performance of the channel estimation algorithm is tested under different Doppler frequencies, modulation schemes, and interpolation methods.

The Kalman filter is proven as an efficient technique to improve the channel estimate without wasting in bandwidth, Where the Kalman filter estimator improved upon the performance of the least square estimator by almost 2-4 dB in term of signal to noise ratio.

However, the performance of the simulated system has been degraded when the Doppler frequency was increased. For a high Doppler frequency, no matter how high we increase the signal to noise ratio (i.e. the power of the transmitter signal), there is always irreducible error rate in the system which called tail error in the fading environment.

In this thesis the comb-type pilot arrangement is adopted which is fit for fast fading channels, in the case of frequency selective channels, it is better to use the block-type pilot arrangement to keep track of the frequency selective channel characteristics. However, when the channel is frequency selective and fast fading the lattice-type pilot arrangement is recommended to be used.

The choice of digital modulation scheme will significantly affect performane Simulation. Results show that the performance of all modulation schemes degrades with the increase of the modulation order. 4QAM modulation is exactly the same performance as QPSK. However, in comparison with PSK, QAM is more power efficient for higher order modulation.

VII. References

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