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# Design and Performance of Horizontal Axis Wind Turbine Using Blade Element Momentum Theory (BEMT)

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**Abstract.** The design of blade of horizontal axis wind turbine (HAWT) has been conducted using model which developed based on blade element momentum theory. The performance of the turbine has been predicted using the same model. Moreover, the improvements of the model have been suggested. To accomplish the design and predict the performance of horizontal axis wind turbine, numerical code has been implemented; the performance of the turbine with and without improvements has been compared with published date. Based on the results presented more improvements have been suggested

#### Introduction

There is significant number of approaches, and models are developed to predict wind turbine performance. Among those methods CFD has been considered the most effective one. However, CFD based approaches cost much time and need special expertise to implement it.

For less accurate approaches, the blade element theory is oldest and remains to be the most widely used method for predicting wind turbine performance .moreover. The Blade Element Momentum (BEM) codes are widely used in the industry. These codes, which are used to calculate steady-state forces acting on blades, are based on analytical formulae [1].

The blade element momentum theory mainly is the combined between the blade theory and momentum theory. This theory has been developed by Glauert to analyze the airplane propeller performance [2].

The blade element theory assumes that the turbine blade consists from a number of elements, that elements can be treats independently as two dimensional airfoils. To analyze that airfoil using blade element theory the forces and moments are calculated for each element then the summation of those forces and moment are obtained that summation is considered as overall force and moment acting on the blade.

The other part of blade element momentum theory (BEMT) the momentum theory, assumes that wind turbine obtains energy from wind flow; thereby, the flow is subjected to pressure and moment losses. Using momentum theory, induced velocities from the momentum loss can be calculated. These induced velocities can affect the flow over blades and the forces on them. By combining the two theories and using an iterative process, forces and momentum on blades can be obtained [2].

However, the original theory that developed by Glauert is not applicable; many corrections have been added to that theory to adapt it for wind turbine design. the corrections added includes tip loss correction, hub loss correction, Glauert and Buhl empirical corrections, skewed wake correction, and '3D correction' of Snel et al. [2]. For this work the corrections considered are the tip loss correction and the Glauert and Buhl empirical corrections [2],[3],[4].

This paper aims to investigate the ability of blade element momentum theory to predict the performance of moderate size wind turbines, that has been conducted by main of validation of the results of numerical calculation based on blade element momentum theory against the experimental measurements of 3.5 kw (UAE) phase VI turbine.

## **Mathematical Derivation**

## Momentum theory

Based on the momentum theory the incremental axial/thrust force  $dF_X$  and angular torque dT on blade sectors can be obtained by applying the conservation of axial momentum and the conservation of angular momentum theory on the blade element, that will yield to the following equations

$$dF_X = \rho U^2 \{4a(1-a)\}\pi r dr \tag{1}$$

$$dT = 4\dot{\alpha}(1-\alpha)\rho U\Omega r^3 \pi dr \tag{2}$$

When a and  $\dot{a}$  are the axial and tangential induction factors where  $\rho$  is the air density, U is the mean air flow velocity, r is the local blade radius,  $\omega$  is the blade angular rotational speed,

The induction factors have been defined as following

The axial induction factor 
$$a = \frac{V_1 - V_2}{V_1}$$
 (3)

The velocities that appear in the equation of induction factor is illustrated in Fig. 1

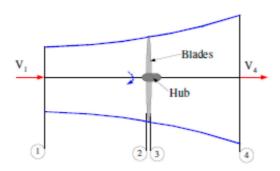


Figure 1 The velocities configuration for linear momentum theory

The angular induction factor 
$$\dot{a} = \frac{\omega}{2\Omega}$$
 (4)

## Blade element theory

Consider a blade divided up into N elements as shown in Fig.2. Each of the blade elements will experience a slightly different flow as they have a different rotational speed (or), a different chord length (c) and a different twist angle (β). Blade element theory involves dividing up the blade into a sufficient number (usually between ten and twenty) of elements and calculating the flow at each one.

Overall performance characteristics are determined by numerical integration along the blade

Based on that theory axial/thrust force  $dF_X$  and angular torque dT on blade sectors can be obtained from the following equations

$$dF_{\chi} = \sigma \pi \rho \frac{V^{2}(1-a)^{2}}{\sin^{2} \phi} (C_{L} \cos \phi + C_{D} \cos \phi) r dr$$

$$dT = \sigma \pi \rho \frac{V^{2}(1-a)^{2}}{\cos \beta} (C_{L} \cos \beta - C_{D} \sin \beta) r^{2} dr$$
(6)

$$dT = \sigma \pi \rho \frac{V^2(1-a)^2}{\cos \beta} \left( C_L \cos \beta - C_D \sin \beta \right) r^2 dr \tag{6}$$

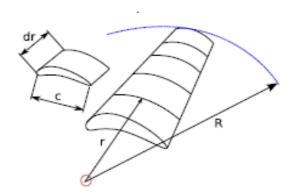


Figure 2 The Blade Element Model

Eventually, the wind turbine power prediction lies in solving the axial induction factor a and the tangential induction factor a [4]. By combining Eq.1, Eq. 2,Eq 5, and Eq. 6, one can solve the induction factors a and a as given below:

$$a = \frac{1}{\frac{4\sin^2\emptyset}{\sigma C_n} + 1} \tag{7}$$

$$a = \frac{1}{\frac{4\sin\phi\cos\phi}{\sigma C_t} - 1} \tag{8}$$

Moreover, the useful relations and terms should be introduced The solidity

$$\sigma = \frac{Bc}{2\pi r} \tag{9}$$

The angle of relative velocity  $\emptyset$ 

$$tan\emptyset = \frac{\lambda(1+\dot{a})}{(1-a)} \tag{10}$$

The tip speed ratio  $\lambda$ 

$$\lambda = \frac{\Omega r}{U} \tag{11}$$

B The number of blades which is 3 for the case studied

The blade geometry and analyses has been illustrated in Fig.3.

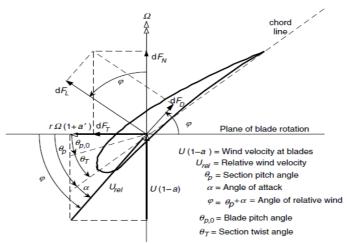


Figure 3 Blade Geometry for the analysis of a HAWT Rotor [4]

For more details about the mathematical derivation of the blade element momentum theory the reader is referred to references [1], [2], [4].

## Corrections of model based on blade element momentum

The correction that will be taken in the account in this study is tip loss correction and, Glauert and Buhl empirical corrections

## **Prandtl Tip Loss Corrections**

Also can be called tip loss correction, this correction comes from the derivation of blade element momentum theory which is based on the assumption the rotor has infinite number of blades. However the real rotor has infinite number of blades real blades of finite length have blade tips. The flow about the ends of the blades changes the wake rotation. This change in the wake rotation has the effect of reducing rotor performance [1].

This correction can be formulated as next

$$F = \frac{\pi}{2} COS^{-1}(\exp(-f))$$
 (12)

$$f = \frac{B(R-r)}{2r\sin\varphi} \tag{13}$$

Thus, the Eq. 1, Eq.2, Eq.7 and Eq. 8 should be corrected and the new formulation of these equations will be

$$dF_X = 4F\rho U^2 \{a(1-a)\}\pi r dr \tag{14}$$

$$dT = 4F4\dot{a}(1-a)\rho U\Omega r^3\pi dr \tag{15}$$

$$a = \frac{1}{\frac{4F\sin^2\phi}{\sigma C_n} + 1} \tag{16}$$

$$\hat{a} = \frac{1}{\frac{4F\sin\phi\cos\phi}{\sigma c_t} - 1} \tag{17}$$

## Glauert and Buhl empirical corrections

The BEM theory is valid only for axial induction factor less than 0.4, if the induction factor is greater than 0.4 wind turbines will be under a turbulence wake.

Glauert developed a correction to the rotor thrust coefficient based on experimental measurements of helicopter rotors with large induced velocities [2].

According to Glauert and Spera, we can further modify Eq.1, so that there are two options should be considered for numerical analysis [1]:

1. When a is less than some critical value, denoted ac, which is taken to equal 0.2, i.e. a < ac, where ac = 0.2:

$$a = \frac{1}{\frac{4F\sin^2\emptyset}{\sigma C_n} + 1}$$

2. When a > ac:

$$a = \frac{1}{2} \left[ 2 + K(1 - 2a_c) - \sqrt{(K(1 - a_c) + 2)^2 + 4(Ka_c^2 - 1)} \right]$$
 (18)

$$K = \frac{4F\sin^2\phi}{\sigma C_n} \tag{19}$$

## Aerodynamics of the Turbine Blade

The airfoil that has been used for the blades of wind turbine mentioned is S809 airfoil the aerodynamic data for S809 airfoil were taken from [5]. The airfoil aerodynamic data, as shown in Fig. 4, were initially developed at TU-Delft wind tunnel, the shape of the airfoil is shown in Fig. 5

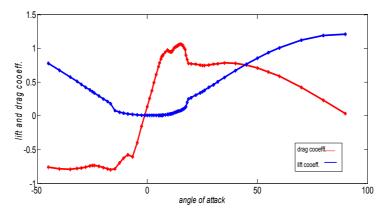


Figure 4 The airfoil aerodynamic data

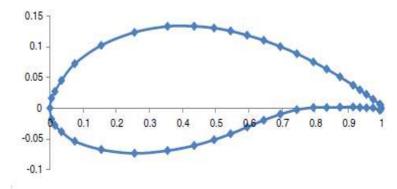


Figure 5 S809 airfoil shape [2]

## **Results and Discussions**

Fig. 6 shows the local twist distribution and chord distribution for the designed rotor blades.

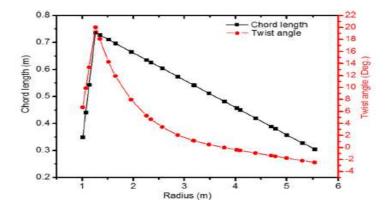


Figure 6 Chord lengths and twist angle over blade [2]

#### Validation of the corrected BEM model

To meet the purpose of this work the numerical results has been validated against small size HAWT, 3.5-kW three-bladed turbine with a 5.532-m rotor diameter.

Fig. 7 shows the power coefficient at different wind speed it is noticed that the highest power coefficient which is 0.47 can be obtained at 7.5 m/s wind speed.

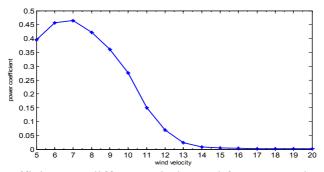


Figure 7 Power coefficients at different wind speed for corrected BEMT model based.

The comparison of the results of the model devolved based on BEMT with the corrected model based on BEMT and the experimental date that has been taken from ref.2 is illustrated in Fig. 8.

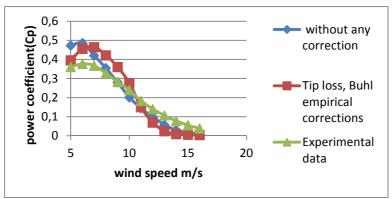


Figure 8 Power coefficients under different models

From the figure mentioned above the gap between the experimental results and the results that obtained from the un-corrected BEMT model can be observed. Therefore, the BEMT without any corrections overestimate the performance of the horizontal axis wind turbine. Moreover, the

discrepancies between the results obtained from the model with tip loss and Buhl empirical correction and the experimental data can still be seen. In this case the mismatching can be referred mainly to the other effects that should be taken in the account such as rotational effect and skewed wake effect.

While the main disadvantage of the rotational effect is that the wind turbine tends to exceed the design value of power, the effect of skewed wake appear because that the BEMT method is suggested for axsymmetric flow but the wind turbine usually running at yaw angle [2].

## **Summary**

As indicated in the last section it seems that the developed model based on the blade element momentum theory with two corrections illustrated has predict the performance of the horizontal axis wind turbine. However, more enhancements should be added to the model to fulfillment the horizontal axis wind turbine design and performance prediction.

#### References

- [1] Niall McMahon, More about Aerodynamics, center of renewable energy at DkIT, 2013.
- [2] Su Liu, Isam Janajreh Development and application of an improved blade element momentum model on horizontal axis wind turbines International Journal of Energy and Environmental Engineering 2012.
- [3] Emrah Kulunk, Nadir Yilmaz, HAWT Rotor Design and Performance Analysis, proceedings of the ASME 2009 3<sup>th</sup> international conference of energy sustainability ES2009.
- [4] J.F. Manwell, J.G. McGowan and A.L.Rogres, Wind energy explained theory, design and application, John Wiley and sons Ltd, 2002.
- [5] Lindenburg, C: Investigation into Rotor Blade Aerodynamics. ECN, Peten (2003).

## **Power and Energy Systems III**

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