Genetic Algorithm to Construct and Enumerate 4×4 Pan-Magic Squares

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***Abstract*—**Since 2700 B.C the problem of constructing magic squares attracts many researchers. Magic squares one of most difficult challenges for mathematicians. In this work, we describe how to construct and enumerate Pan- magic squares using genetic algorithm, using new chromosome encoding technique. The results were promising within reasonable time.

***Keywords*—**Genetic algorithm, Magic square, Pan-magic square.

I. Introduction

THE problem of constructing and enumerate magic squares is extremely difficult to solve and easy to understand. This problem has many applications in mathematics such as theories of groups, matrices, etc. Magic square can be classified as optimization problem since we looking for the optimal arrangement of given positive numbers to fit some constrains, therefore Genetic algorithm can be one of best method to solve this problem. In this work, we use genetic algorithm to constructing and enumerate 4×4 pan-magic squares, using novel and simple encoding and fitness function. We are looking forward to improving this algorithm to solve higher order magic squares.

II. Magic Square

A magic square is a square array $n \times n$ of numbers consisting of the distinct positive integers $1, 2, \ldots, 2^n$ arranged such that the sum of the n numbers in any column, row or both diagonals is equal to the magic constant M. [1] If all the diagonals including broken diagonals sum to M then the magic square is said to be pan-diagonal
where

$$M = \frac{1}{2}n(n^2 + 1)$$

Magic squares of order N are composed of the entries $1, 2, \ldots, 2^n$ arranged on a square unit lattice such that the sum of all entries along the rows, columns and main diagonals are equal to the magic constant of the square. An example of a magic square is shown in Fig. 1.

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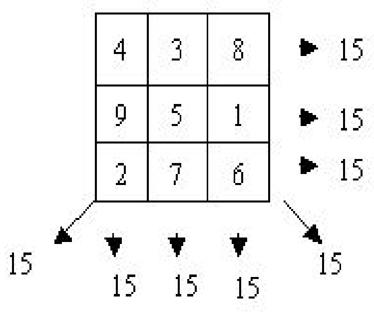

Fig. 1 3×3 magic square

III. Pan-Magic square

There are The 384 Pan-Magic Squares **If X is 4**×4 magic square and also the sum pan diagonals elements is 34

1	8	13	12
14	11	2	7
4	5	16	9
15	10	3	6

Fig. 2 4×4 pan- magic square

In 4×4 pan-magic squares, the magic constant of 34 can be seen in a number of patterns in addition to the rows, columns and diagonals [2]:

- Any of the sixteen 2×2 squares, including those that wrap around the edges of the whole square, e.g. 14+11+4+5, 1+12+15+6 and so on
- The corners of any 3×3 square, e.g. 8+12+5+9
- Any pair of horizontally or vertically adjacent numbers, together with the corresponding pair displaced by a (2, 2) vector, e.g. 1+8+16+9

In any 4×4 pan diagonal magic square, any two numbers at the opposite corners of a 3×3 square add up to 17. Consequently
X(1,1)+X(3,3) ; X(1,2)+X(3,4); X(1,3)+X(3,1); X(1,4)+X(3,2)
X(2,1)+X(4,3); X(2,2)+X(4,4); X(2,3)+X(4,1); X(2,4)+X(4,2)

IV. Genetic Algorithm

Genetic algorithm (GA) is adaptive heuristic search algorithm inspired on the evolutionary ideas of natural selection and survival of the fittest, [3] where each chromosome in the population represents candidate solution for the problem, each two chromosomes are recombined with crossover operation to produce offspring for new generation, mutation is applied to some of the new generation according to mutation rate, each the individuals in the new population is evaluated depending on its fitness value therefore the fittest ones will be selected to produce a new generation. Genetic algorithms are used to solve optimization problem which is NP-hard, the magic square problem *can* be represented as a permutation problem.

A. Encoding

The most difficulty in using a genetic algorithm is finding an appropriate encoding of the problem solutions to a chromosome. A good encoding of the chromosome can limiting the search space, therefore the search become simple and easy. In this work we represents 4 × 4 magic square as GA chromosome by one dimension array 1 X 16 where the first four digits of the chromosome represented the first row; the second four digits represented the second row, and so on. For example the following magic square

16	2	3	13
5	11	10	8
9	7	6	12
4	14	15	1

Fig. 3 4×4 magic square

Will be represented as the following chromosome:

16 2 3 13 5 11 10 8 9 7 6 12 4 14 15 1

B. Fitness Function

Fitness function is the way to evaluate the solutions which represented as chromosomes, therefore according to the properties of pan magic square we designed the fitness function for GA in order to solve the problem of pan diagonal magic square, and the fitness function ***f*** can be calculated as following

$$f=\left(\sum_{i=1}^{4}|C_i-m|\right)+\left(\sum_{i=1}^{4}|R_i-m|\right)+\left(\sum_{i=1}^{2}|D_i-m|\right)$$

where C_i is the column i, R_i is the row i, D_i is diagonals and m is the magic constant.

If $\mathbf{f}=\mathbf{0}$ then the square is fit to be magic square therefore the fitness function checks the rest of pan-magic constrains such as the sum pan diagonals elements is 34, otherwise it is not magic square.

C. Rank Selection

Genetic algorithm chromosomes are evaluated according to a fitness function, to select the fittest ones for a recombination process, which produces new chromosomes hopeful of having a fitter offspring; we use rank selection to select the best chromosomes; where the population is ranked according to their fitness values.

D. Crossover

Crossover is the most important operator for GA, therefore one have to use the suitable one to ensure finding optimal result in reasonable time. According to the IGA the crossover operation used in this work is Swapped Inverted Crossover (SIC) [4], which have proven to be effective for solving the magic square problem. In this operation, the selected two chromosomes are recombined using different ways to produce new 100 chromosomes inherited the good characteristics from their parents, therefore, the new generation will be better than the previous one.

E. Mutation

In this work two mutation operations; are used first one is swapping between two random bits, if the chromosome is 6 14 8 10 4 16 **5** 3 12 9 1 13 11 **7** 2 15 then after swapped between the two bits 5 and 7 the chromosome became

6 14 8 10 4 16 **7** 3 12 9 1 13 11 **5** 2 15

Second mutation technique is swapping four adjacent bits by another four adjacent bits also selected by random way, if the chromosome is

6 14 **8 10 4 16** 7 3 12 **9 1 13 11** 5 2 15

After second mutation it will be

6 14 **9 1 13 11** 7 3 12 **8 10 4 16** 5 2 15

V. Conclusion and Future Work

In this work, we implement the genetic algorithm using Matlab to construct and enumerate all 4×4 Pan- magic squares and the results were promising in reasonable time. This algorithm can be modified to solve high order magic squares such as 5×5 or 6×6.

References

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