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ON GENERAL INTEGRAL OPERATOR DEFINED BY LINEAR DERIVATIVE OPERATOR

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Abstract

The purpose of the present paper we consider some sufficient conditions for new general integral operators of p- valent functions on the new classes $US_p^{\alpha,\delta}(\mu,q,\lambda,k,\beta)$ and $UC_p^{\alpha,\delta}(\mu,q,\lambda,k,\beta)$ to be convex functions defined in the open unit disk.

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1 Introduction

let A_p denote the class of functions of the form :

$$f(z) = z^p + \sum_{k=p+1}^{\infty} a_k z^k, \qquad (p \in \mathbb{N}).$$

which are analytic in the open unit disk $\mathbb{U} = \{z : |z| < 1\}$. A function f belonging to A_p is said to be p-valently starlike of order β if it satisfies

$$Re\left\{\frac{zf'(z)}{f(z)}\right\} > \beta \quad (z \in \mathbb{U}),$$

for some $\beta \quad (0 \leq \beta < p)$. We denote by $S_p^*(\beta)$ the subclass of A_P consisting of functions which are p-valently starlike of order β in $\mathbb{U}$.
Further, a function f belonging to A_p is said to be p-valently convex of order β if it satisfies

$$Re\left\{1+\frac{zf''(z)}{f'(z)}\right\} > \beta \quad (z \in \mathbb{U}),$$

for some $\beta \quad (0 \leq \beta < p)$. We denote by $C_p(\beta)$ the class of functions in A_p which are p-valently convex of order β in $\mathbb{U}$. A function $f \in A_p$ is said to be in

the class $US_p(k,\beta)$ of k-uniformly p-valent starlike of order $\beta \quad (-1 \leq \beta < p)$. in $\mathbb{U}$ and satisfies

$$Re\left\{\frac{zf'(z)}{f(z)}-\beta\right\} \geq k\left|\frac{zf'(z)}{f(z)}-p\right| \quad (k \geq 0, \quad z \in \mathbb{U}),$$

Furthermore, a function $f \in A_p$ is said to be in the class $UC_p(k,\beta)$ of k-uniformly p-valent convex of order $\beta \quad (-1 \leq \beta < p)$. in $\mathbb{U}$ and satisfies

$$Re\left\{1+\frac{zf''(z)}{f'(z)}-\beta\right\} \geq k\left|1+\frac{zf''(z)}{f'(z)}-p\right| \quad (k \geq 0, \quad z \in \mathbb{U}).$$

Note that $US_1(k,\beta) = US(k,\beta)$ and $UC_1(k,\beta) = UC(k,\beta)$ where the classes

$US(k,\beta)$ and $UC(k,\beta)$ are, respectively, the classes k-uniformly starlike of order $\beta \quad (0 \leq \beta < p)$ and k-uniformly convex of order studied in (A.W. Goodman,1991).
Now, $(x)_k$ denotes the Pochhammer symbol (or the shifted factorial) defined by

$$(x)_k = \begin{cases} 1 & for \quad k = 0, x \in \mathbb{C} - \{0\}, \\ x(x+1)(x+2)...(x+k-1) & for \quad k \in \mathbb{N} = 1,2,3,...and \quad x \in \mathbb{C}. \end{cases}$$

We state a generalization linear derivative operator $D_p^{\alpha,\delta}(\mu,c,\lambda)$ given as the following (2010):

Definition 1.1 *For $f \in A_p$ and $\lambda, q, \mu \geq 0$ the linear operator $D_p^{\alpha,\delta}(\mu,q,\lambda)$ is defined by $D_P^{\alpha,\delta}(\mu,q,\lambda) : A_p \to A_p$ as*

$$D_P^{\alpha,\delta}(\mu,q,\lambda) = z^p + \sum_{k=p+1}^{\infty} k^{\alpha}(1+\frac{k-p}{p+q}\lambda)^{\mu} c(\delta,k) a_k z^k,$$

where $c(\delta,k) = \frac{(\delta+1)_{k-1}}{(1)_{k-1}}$.

Definition 1.2 *For* $f \in A_p$, $(-1 \leq \beta < p)$ *and* $\lambda, q, \mu \geq 0$ *let* $US_p^{\alpha,\delta}(\mu, q, \lambda, k, \beta)$ *and* $UC_p^{\alpha,\delta}(\mu, q, \lambda, k, \beta)$ *be the subclasses satisfying*

$$Re\left\{\frac{z(D_p^{\alpha,\delta}(\mu,q,\lambda)f(z))'}{D_p^{\alpha,\delta}(\mu,q,\lambda)f(z)}-\beta\right\} \geq k\left|\frac{z(D_p^{\alpha,\delta}(\mu,q,\lambda)f(z))'}{D_p^{\alpha,\delta}(\mu,q,\lambda)f(z)}-p\right| \quad (k \geq 0, z \in \mathbb{U}),$$

and

$$Re\left\{1+\frac{z(D_p^{\alpha,\delta}(\mu,q,\lambda)f(z))''}{(D_p^{\alpha,\delta}(\mu,q,\lambda)f(z))'}-\beta\right\} \geq k\left|1+\frac{z(D_p^{\alpha,\delta}(\mu,q,\lambda)f(z))''}{(D_p^{\alpha,\delta}(\mu,q,\lambda)f(z))'}-p\right| \quad (k \geq 0, z \in \mathbb{U}),$$

respectively.

Note that $US_1^{0,0}(0,q,\lambda,k,\beta)$=$US(k,\beta)$ and $UC_1^{0,0}(0,q,\lambda,k,\beta)$=$UC(k,\beta)$

Definition 1.3 *For* $f_i \in A_p$ *and* $\gamma_i > 0$ *we define the following general integral operators*

$$D_p^{\alpha,\delta}(\mu,q,\lambda)F_p(z) = \int_0^z pt^{p-1}(\frac{D_p^{\alpha,\delta}(\mu,q,\lambda)f_1(z)}{t^p})^{\gamma_1}.....(\frac{D_p^{\alpha,\delta}(\mu,q,\lambda)f_n(z)}{t^p})^{\gamma_n}dt$$

and

$$D_p^{\alpha,\delta}(\mu,q,\lambda)G_p(z) = \int_0^z pt^{p-1}(\frac{D_p^{\alpha,\delta}(\mu,q,\lambda)f_1(z)}{pt^{p-1}})^{\gamma_1}....(\frac{D_p^{\alpha,\delta}(\mu,q,\lambda)f_n(z)}{pt^{p-1}})^{\gamma_n}dt$$

Corollary 1.4 *It is interesting to note that the integral operators generalizes many operators* $D_p^{\alpha,\delta}(\mu,q,\lambda)F_p(z)$ *and* $D_p^{\alpha,\delta}(\mu,q,\lambda)G_p(z)$ *which were introduced and studied recently.*

• *When* $p = 1$, $\delta = \alpha = \mu = 0$ *the operator* $D_p^{\alpha,\delta}(\mu,q,\lambda)F_p(z)$ *reduces to an integral operator*

$$F_n(z) = \int_0^z (\frac{f_1(z)}{t})^{\gamma_1} \cdots\cdots (\frac{f_n(z)}{t})^{\gamma_n}dt$$

introduced and studied by (Breaz, N. Breaz, 2002)and (D. Breaz, S. Owa, N. Breaz,2008).

• *When* $p = n = 1$, $\delta = \alpha = \mu = 0$ *and* $\gamma_1 = \gamma \in [0,1]$ *the operator* $D_p^{\alpha,\delta}(\mu,q,\lambda)F_p(z)$ *reduces to an integral operator* $\int_0^z (\frac{f(z)}{t})^{\gamma}dt$ *studied in (S.S. Miller, P.T. Mocanu, ,1978).*

• *When* $p = 1$, $\delta = \alpha = \mu = 0$ *the operator* $D_p^{\alpha,\delta}(\mu,q,\lambda)G_p(z)$ *reduces to an integral operator*

$$F_{\gamma_1,\gamma_2....\gamma_n}(z) = \int_0^z (f_1'(z))^{\gamma_1} \cdots\cdots (f_n'(z))^{\gamma_n}dt$$

introduced and studied by (D.Breaz, N. Breaz, 2002)and (D. Breaz, S. Owa, N. Breaz,2008).

• *When* $p = n = 1, \quad \delta = \alpha = \mu = 0$ *and* $\gamma_1 = \zeta \in \mathbb{N}$, $(|\zeta| < \frac{1}{4})$ *the operator* $D_p^{\alpha,\delta}(\mu, q, \lambda)G_p(z)$ *reduces to an integral operator* $\int_0^z f'(z)^\gamma dt$ *studied in studied (N.Pascu, V. Pescar,1990).*

2 Main result

Theorem 2.1 *For* $\gamma_i > 0, -1 \leq \beta_i < p$ *and* $k_i > 0$ *for all* $i = 1, 2, 3.....n$ $US_p^{\alpha,\delta}(\mu, q, \lambda, k_i, \beta_i)$ *for all* $i = 1, 2, 3.....n$ *.If* $0 \leq p + \sum_{i=1}^n \gamma_i(\beta_i - p) < 0$. *Then the integral operator* $D_p^{\alpha,\delta}(\mu, q, \lambda)F_p(z)$ *is p- valently convex of order* $p + \sum_{i=1}^n \gamma_i(\beta_i - p)$.

Remark 2.2 *Letting* $p = 1, \quad \mu = \alpha = \delta = 0 \quad , \beta_i = 0$ *for all* $i = 1, 2, 3.....n$ *in* 2.1 *we get Theorem 2.5 (D. Breaz, N. Breaz,2006.)*
Letting $p = n = 1$, $\mu = \alpha = \delta = 0$, $\beta_i = \beta$,$\gamma_i = \gamma$, $\quad k_i = k$ *and* $f_i = f$ *in* 2.1, *then we have*

Corollary 2.3 *Let* $\gamma > 0, -1 \leq \beta < p$ *and* $US_p^{\alpha,\delta}(\mu, q, \lambda, k, \beta)$ *.If* $0 \leq 1 + \gamma(\beta - p) < 0$. *Then the integral operator* $D_p^{\alpha,\delta}(\mu, q, \lambda)F_p(z)$ *is p- valently convex of order* $1 + \gamma(\beta - p)$.

Theorem 2.4 *For* $\gamma_i > 0, -1 \leq \beta_i < p$ *and* $k_i > 0$ *for all* $i = 1, 2, 3.....n$ $UC_p^{\alpha,\delta}(\mu, q, \lambda, k_i, \beta_i)$ *for all* $i = 1, 2, 3.....n$ *.If* $0 \leq p + \sum_{i=1}^n \gamma_i(\beta_i - p) < 0$. *Then the integral operator* $D_p^{\alpha,\delta}(\mu, q, \lambda)G_p(z)$ *is p- valently convex of order* $p + \sum_{i=1}^n \gamma_i(\beta_i - p)$.

References

[1] *Far East J. Math. Sci.* **15**, (2004),n0.1, 87-94.

[2] S. S. Miller and P. T. Mocanu, "Subordinants of differential superordinations, *Indag. Math. (N. S.)*, **13** , (2003),no.10, 815-826.