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UNIVALENCE CONDITIONS FOR A NEW GENERAL INTEGRAL OPERATOR

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ملخص

في هذا البحث، نقوم بتعريف مشغل تكاملي جديد في قرص الوحدة المفتوحة. والهدف الرئيسي من هذه الورقة هو الحصول على شروط احادية التكافؤ لهذا المشغل التكاملي أيضا العديد من النتائج لأخرى ذات الصلة .

Abstract

In this paper, we define a new general integral operator in the open unit disk U. The main object of this paper is to obtain new sufficient conditions for the univalence of this general integral operator. Several corollaries of the main results are also considered.

Introduction:

Let A denote the class of functions of the form:

$$f(z)=z+\sum_{k=2}^{\infty}a_k z^k, \qquad (1)$$

which are analytic in the open unit disk $\mathrm{U}=\{z\in\mathrm{C}:|z|<1\}$. Further, by S we shall denote the class of all functions in A which are univalent in U. For two functions, $f(z)\in\mathrm{A}$ and $g(z)$ given by

$$g(z)=z+\sum_{k=2}^{\infty}b_k z^k,$$

their convolution (or Hadamard product) is defined by

$$(f*g)(z)=z+\sum_{k=2}^{\infty}a_k b_k z^k.$$

The theory of derivative and integral plays an important role in the theory of univalent functions. It is believed that Ruscheweyh (1975) was the first to give a generalised derivative operator in the theory of univalent function. Later, S‹al‹agean (1983) gave another generalised derivative operator. In the same paper, he introduced an integral operator. Many properties have been discussed and studied by many researchers for these two operators. For example, Al-Oboudi (2004) introduced a generalised Salagean operator, Al-Shaqsi and Darus (2009) generalised the operator given by Ruscheweyh (1975), while Darus and Al-Shaqsi (2008) studied both derivatives of Ruscheweyh and Salagean. These operators motivate us to create another type of derivative operator.

The author in [1] have recently introduced a new generalised derivative operator $\mathrm{D}^{\alpha,n}(m,q,\lambda)f(z)$ as the following:

For the function $f\in\mathrm{A}$ given by (1) we define a new generalised derivative operator $\mathrm{D}^{\alpha,n}(m,q,\lambda)f(z):\mathrm{A}\to\mathrm{A}$ as follows:

$$\mathrm{D}^{\alpha,n}(m,q,\lambda)(f)(z)=z+\sum_{k=2}^{\infty}k^{\alpha}\left(1+\frac{k-1}{1+q}\lambda\right)^{m}c(n,k)a_k z^k, \qquad (1.2)$$

where $n,\alpha\in\mathrm{N}_0=\{0,1,2...\},m\in\mathrm{Z},\ \lambda,q\geq 0$ and $c(n,k)=\frac{(n+1)_{k-1}}{(1)_{k-1}}$.

Here $\mathrm{D}^{\alpha,n}(m,q,\lambda)f(z)$ can also be written in terms of convolution as

$$\phi(z):=\left(\frac{1+q-\lambda}{1+q}\right)\frac{z}{1-z}+\left(\frac{\lambda}{1+q}\right)\frac{z}{(1-z)^2}, \quad (z\in\mathrm{U}).$$

If $m=0,1,2,...$, then

$$\mathrm{D}^{\alpha,n}(m,q,\lambda)f(z)=\underbrace{\phi(z)*...*\phi(z)}_{(m)-times}*\left[\frac{z}{(1-z)^{n+1}}\right]*\sum_{k=1}^{\infty}k^{\alpha}z^k*f(z)$$

$$=R^n*\mathrm{D}^{\alpha}(m,q,\lambda)f(z),$$

where $R^n=z+\sum_{k=2}^{\infty}c(n,k)z^k$, the Ruscheweyh derivative operator.

If $m=-1,-2,...$, then

$$D^{\alpha,n}(m,q,\lambda)f(z)=\underbrace{\phi(z)*...*\phi(z)}_{(-m)-times}*\left[\frac{z}{(1-z)^{n+1}}\right]*\sum_{k=1}^{\infty}k^{\alpha}z^{k}*f(z)$$

$$=R^{n}*D^{\alpha}(m,q,\lambda)f(z).$$

Note that:

$$D^{0,0}(0,q,\lambda)f(z)=D^{0,0}(1,0,0)f(z)=f(z),\quad and$$

$$D^{0,0}(1,q,\lambda)f(z)=zf'(z).$$

By specialising the parameters of $D^{\alpha,n}(m,q,\lambda)f(z)$, we get the following derivative and integral operators.

- The derivative operator introduced by Ruscheweyh [2];

$$D^{0,n}(0,q,\lambda)\equiv D^{0,n}(1,0,0);(n\in N_0)\equiv R^{n}=z+\sum_{k=2}^{\infty}c(n,k)a_k z^{k}.$$

- The derivative operator introduced by Sǎlǎgean [3];

$$D^{\alpha,0}(0,q,\lambda)\equiv D_1^{0,0}(n,0,1);(n\in N_0)\equiv D^{n}=z+\sum_{k=2}^{\infty}k^{n}a_k z^{k}.$$

- The generalised Salagean derivative operator introduced by Oboudi [4];

$$D^{0,0}(n,0,\lambda);(n\in N_0)\equiv D_{\lambda}^{n}=z+\sum_{k=2}^{\infty}(1+\lambda(k-1))^{n}a_k z^{k}.$$

- The generalised Ruscheweyh derivative operator introduced by Darus and Al-Shaqsi [5];

$$D^{0,n}(1,0,\lambda);(n\in N_0)\equiv R_{\lambda}^{n}=z+\sum_{k=2}^{\infty}(1+\lambda(k-1))c(n,k)a_k z^{k}.$$

- The derivative operator introduced by Catas [6];

$$D^{0,\beta}(m,l,\lambda);(m\in N_0)\equiv D^{m}(\lambda,\beta,l)=z+\sum_{k=2}^{\infty}\left(\frac{1+\lambda(k-1)+l}{1+l}\right)^{m}c(\beta,k)a_k z^{k}.$$

- The integral operator introduced by Cho and T. H. Kim [7];

$$D^{1,0}(-n,\lambda,1)\equiv I_n^{\lambda}=z+\sum_{k=2}^{\infty}k\left(\frac{1+\lambda}{k+\lambda}\right)^{n}a_k z^{k}.$$

The study of integral operators has been rapidly investigated by many authors, the Alexander transformation (Alexander in 1915), Libera integral operator (Libera 1965)
and later the Bernardi integral operator (Bernardi 1969). By using the generalised derivative operator given by Definition 1.2, we introduce the following integral operator.

Definition 1.1 *For* $f_i\in A$, $i=\{1,2,3,\ldots,s\}$, $n\in N\cup\{0\}$ *and* $\gamma_1,\gamma_2,\gamma_3,\ldots,\gamma_s,\in C$, *we define a family of integral operator by*

$$F^{\alpha,n}(m,q,\lambda,\gamma_i;z)=\int_0^z\prod_{i=1}^{s}\left(\frac{D^{\alpha,n}(m,q,\lambda)f_i(z)}{z}\right)^{\frac{1}{\gamma_i}}dt \qquad (1.3)$$

where $\alpha\in N_0=\{0,1,2...\}, m\in Z,\ \lambda,q\geq 0$ and $D^{\alpha,n}(m,q,\lambda)$ defined by (1.2), which generalises many integral operators. In fact, if we choose suitable values of parameters, we get the following interesting operators.

1. $m=0,\ \alpha=0,\ n=0,\ \gamma_i=\dfrac{1}{\alpha-1}$, we obtain $F_{n,\alpha}(z)$ of by Breaz, Breaz and Srivastava [10].

2. $m=0,\ \alpha=0,\ n=0,\ \gamma_i=\dfrac{1}{\alpha_i}$, we obtain $F_n(z)$ of by Breaz and Breaz [9].

Recently many authors (see for example [2, 9, 10] and [11] have studied and obtained univalence conditions for the analytic function.

In the present paper, we also obtain univalence conditions for integral operator which is defined by (1.3).
To prove our main results we need followings Lemmas.

Lemma 1.2 *[12] If the function f is regular in the unit disk* U,
and satisfies the inequality

$$(1-|z|^2)\left|\frac{zf''(z)}{f'(z)}\right|\leq 1 \qquad \text{for all} \quad z\in \mathrm{U},$$

then the function f is univalent in U.

Lemma 1.3 *[13] (Schwarz's Lemma)*
If the analytic function $f(z)$ is regular in U, with $f(0)=0$ and $|f(z)|<1$, for all $z\in\mathrm{U}$, then

$$|f(z)|<|z| \qquad \forall z\in\mathrm{U},\ \text{and}\ |f'(0)|\leq 1.$$

The equality holds if and only if $f(z)=cz, |c|=1, \quad z\in\mathrm{U}$.

Main results

Our main result is a application of Lemma 1.2 and contains sufficient conditions for an general integral operator $\mathrm{F}^{\alpha,n}(m,q,\lambda,\gamma_i;z)$

Theorem 2.1 *For $f_i\in\mathrm{A}$, $i=\{1,2,3,\ldots,s\}$, $n\in\mathrm{N}\cup\{0\}$ and $\gamma_1,\gamma_2,\gamma_3,\ldots,\gamma_s,\in\mathrm{C}$ If*

$$\left|\frac{z(\mathrm{D}^{\alpha,n}(m,q,\lambda)f_i(z))'}{\mathrm{D}^{\alpha,n}(m,q,\lambda)f_i(z))}-1\right|\leq 1,$$

and

$$\left|\frac{1}{\gamma_1}\right|+\left|\frac{1}{\gamma_2}\right|+\ldots+\left|\frac{1}{\gamma_s}\right|\leq 1,\quad z\in\mathrm{U}. \tag{2}$$

Then $\mathrm{F}^{\alpha,n}(m,q,\lambda,\gamma_i;z)$ is univalent.

Proof: Since $i\in 1,2,\ldots,s, f_i\in\mathrm{A}$, we have

$$\frac{\mathrm{D}^{\alpha,n}(m,q,\lambda)f_i(z)}{z}=\frac{z+\sum_{k=2}^{\infty}k^{\alpha}(1+\frac{k-1}{1+q}\lambda)^m c(n,k)a_k z^k}{z}, \tag{3}$$

$$=1+\sum_{k=2}^{\infty}k^{\alpha}(1+\frac{k-1}{1+q}\lambda)^{m}c(n,k)a_k z^k a_k z^{k-1}\neq 0,\quad z\in \mathrm{U}.$$

By differentiating 1.3, we obtain

$$[\mathrm{F}^{\alpha,n}(m,q,\lambda,\gamma_i;z)]'=[\frac{\mathrm{D}^{\alpha,n}(m,q,\lambda)f_1(z)}{z}]^{\frac{1}{\gamma_1}}\ldots[\frac{\mathrm{D}^{\alpha,n}(m,q,\lambda)f_s(t)}{z}]^{\frac{1}{\gamma_s}},z\in \mathrm{U}. \quad (4)$$

and we have

$$[\mathrm{F}^{\alpha,n}(m,q,\lambda,\gamma_i;0)]'=\mathrm{F}^{\alpha,n}(m,q,\lambda,\gamma_i;0)=1$$

Also, a simple computation yields Using 4, we obtain

$$\ln[\mathrm{F}^{\alpha,n}(m,q,\lambda,\gamma_i;z)]'=\frac{1}{\gamma_1}[\ln \mathrm{D}^{\alpha,n}(m,q,\lambda)f_1(z)-\ln z]+\ldots+\frac{1}{\gamma_s}[\ln \mathrm{D}^{\alpha,n}(m,q,\lambda)f_s(z)-\ln z], \quad (5)$$

By differentiating 5, we have

$$\frac{[\mathrm{F}^{\alpha,n}(m,q,\lambda,\gamma_i;z)]''}{[\mathrm{F}^{\alpha,n}(m,q,\lambda,\gamma_i;z)]'}=\frac{1}{\gamma_1}[\frac{(D^{\alpha,n}(m,q,\lambda)f_1(z))'}{\mathrm{D}^{\alpha,n}(m,q,\lambda)f_1(z)}-\frac{1}{z}]+\ldots+\frac{1}{\gamma_s}[\frac{(\mathrm{D}^{\alpha,n}(m,q,\lambda)f_s(z))'}{\mathrm{D}^{\alpha,n}(m,q,\lambda)f_s(z)}-\frac{1}{z}],\quad z\in \mathrm{U}. \quad (6)$$

Simple computation, we get

$$(1-|z|^2)\left|\frac{z[\mathrm{F}^{\alpha,n}(m,q,\lambda,\gamma_i;z)]''}{[\mathrm{F}^{\alpha,n}(m,q,\lambda,\gamma_i;z)]'}\right|$$

$$=(1-|z|^2)\left|\frac{1}{\gamma_1}[\frac{z(\mathrm{D}^{\alpha,n}(m,q,\lambda)f_1(z))'}{\mathrm{D}^{\alpha,n}(m,q,\lambda)f_1(z)}-1]+\ldots+\frac{1}{\gamma_s}[\frac{z(\mathrm{D}^{\alpha,n}(m,q,\lambda)f_s(z))'}{\mathrm{D}^{\alpha,n}(m,q,\lambda)f_s(z)}-1]\right|,$$

$$\leq(1-|z|^2)\left[|\frac{1}{\gamma_1}|\left|\frac{z(\mathrm{D}^{\alpha,n}(m,q,\lambda)f_1(z))'}{\mathrm{D}^{\alpha,n}(m,q,\lambda)f_1(z)}-1\right|+\ldots+|\frac{1}{\gamma_1}|\left|\frac{z(\mathrm{D}^{\alpha,n}(m,q,\lambda)f_s(z))'}{\mathrm{D}^{\alpha,n}(m,q,\lambda)f_s(z)}-1\right|\right],$$

$$\leq(1-|z|^2)[|\frac{1}{\gamma_1}|+|\frac{1}{\gamma_2}|+\ldots+|\frac{1}{\gamma_s}|]\leq|\frac{1}{\gamma_1}|+|\frac{1}{\gamma_2}|+\ldots+|\frac{1}{\gamma_s}|\leq 1.$$

Thus by Lemma 1.2, we have $\mathrm{F}^{\alpha,n}(m,q,\lambda,\gamma_i;z)$ is univalent.

Setting $m=0,\ \alpha=0,\ n=0,\ \gamma_i=\frac{1}{\alpha_i}$ in Theorem 2.1, we obtain the following consequence of Theorem

Corollary 2.1 *For* $f_i\in \mathrm{A}\ \ i=\{1,2,3,\ldots,s\}$, *If*

$$\left|\frac{z((f_i)(z))'}{(f_i)(z)}-1\right|=\left|\frac{z(F^{0,0}(0,q,\lambda,\gamma_i;z))'}{F^{0,0}(0,q,\lambda,\gamma_i;z)}-1\right|\leq 1, \text{ and}$$

$$|\alpha_1|+|\alpha_2|+...+|\alpha_s|\leq 1, z\in U.$$

Then $(f_i)(z)$ is univalent, where $(f_i)(z)$ is given by Breaz and Breaz [9].

Theorem 2.2 *For* $i=\{1,2,3,L\ ,s\}$, $n\in N\cup\{0\}$ *and* $\gamma_1,\gamma_2,\gamma_3,L\ ,\gamma_s,\in C$ *If* $f_i\in A$ *satisfy*

$$(i)\left|\frac{1}{\gamma_1}\right|+\left|\frac{1}{\gamma_2}\right|+...+\left|\frac{1}{\gamma_s}\right|\leq\frac{1}{3},$$

$$(ii)\,|F^{\alpha,n}(m,q,\lambda,\gamma_i;z)|\leq 1,$$

$$(iii)\left|\frac{z^2[F^{\alpha,n}(m,q,\lambda,\gamma_i;z)]'}{F^{\alpha,n}(m,q,\lambda,\gamma_i;z)]^2}-1\right|<1.$$

For all $z\in U$, then the integral operator given by (1.3) is univalent.

Proof: Using 6, we obtain

$$\left|\frac{z[F^{\alpha,n}(m,q,\lambda,\gamma_i;z)]''}{[F^{\alpha,n}(m,q,\lambda,\gamma_i;z)]'}\right|\leq\left|\frac{1}{\gamma_1}\right|\left|[\frac{z(D^{\alpha,n}(m,q,\lambda)f_1(z))'}{D^{\alpha,n}(m,q,\lambda)f_1(z)}-1]\right|+...+\left|\frac{1}{\gamma_s}\right|\left|[\frac{z(D^{\alpha,n}(m,q,\lambda)f_s(z))'}{D^{\alpha,n}(m,q,\lambda)f_s(z)}-1]\right| \quad (7)$$

Multiply 7 by $(1-|z|^2)$, using Schwarz's Lemma and obtain

$$(1-|z|^2)\left|\frac{z[F^{\alpha,n}(m,q,\lambda,\gamma_i;z)]''}{[F^{\alpha,n}(m,q,\lambda,\gamma_i;z)]'}\right|\leq(1-|z|^2)\left|\frac{1}{\gamma_1}\right|\left|[\frac{z(D^{\alpha,n}(m,q,\lambda)f_1(z))'}{D^{\alpha,n}(m,q,\lambda)f_1(z)}-1]\right|+...$$
$$+(1-|z|^2)\left|\frac{1}{\gamma_s}\right|\left|[\frac{z(D^{\alpha,n}(m,q,\lambda)f_s(z))'}{D^{\alpha,n}(m,q,\lambda)f_s(z)}-1]\right|$$
$$\leq(1-|z|^2)\left|\frac{1}{\gamma_1}\right|\left|[\frac{z(D^{\alpha,n}(m,q,\lambda)f_1(z))'}{D^{\alpha,n}(m,q,\lambda)f_1(z)}]\right|+(1-|z|^2)\left|\frac{1}{\gamma_1}\right|+...$$
$$+(1-|z|^2)\left|\frac{1}{\gamma_s}\right|\left|[\frac{z(D^{\alpha,n}(m,q,\lambda)f_s(z))'}{D^{\alpha,n}(m,q,\lambda)f_s(z)}]\right|+(1-|z|^2)\left|\frac{1}{\gamma_s}\right|$$

$$\leq(1-|z|^2)\left[\left|\frac{1}{\gamma_1}\right|\left|[\frac{z(D^{\alpha,n}(m,q,\lambda)f_1(z))'}{D^{\alpha,n}(m,q,\lambda)f_1(z)}]\right|+...+\left|\frac{1}{\gamma_s}\right|\left|[\frac{z(D^{\alpha,n}(m,q,\lambda)f_s(z))'}{D^{\alpha,n}(m,q,\lambda)f_s(z)}]\right|\right]$$
$$+(1-|z|^2)[\ \left|\frac{1}{\gamma_1}\right|+...+\left|\frac{1}{\gamma_s}\right|\ ]$$

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$$= (1-|z|^2)\left[\begin{array}{l} |\frac{1}{\gamma_1}|\left|\frac{z^2[D^{\alpha,n}(m,q,\lambda)f_1(z)]'}{[D^{\alpha,n}(m,q,\lambda)f_1(z)]^2}\right|\left|\frac{(D^{\alpha,n}(m,q,\lambda)f_1(z))}{z}\right| + \ldots \\ + |\frac{1}{\gamma_s}|\left|\frac{z^2[D^{\alpha,n}(m,q,\lambda)f_s(z)]'}{[D^{\alpha,n}(m,q,\lambda)f_s(z)]^2}\right|\left|\frac{(D^{\alpha,n}(m,q,\lambda)f_s(z))'}{z}\right|] \end{array}\right] + (1-|z|^2)[|\frac{1}{\gamma_1}|+\ldots+|\frac{1}{\gamma_s}|]$$

$$\le (1-|z|^2)\left[\begin{array}{l} |\frac{1}{\gamma_1}|\left|\frac{z^2[D^{\alpha,n}(m,q,\lambda)f_1(z)]'}{[D^{\alpha,n}(m,q,\lambda)f_1(z)]^2}\right| + \ldots \\ + |\frac{1}{\gamma_s}|\left|\frac{z^2[D^{\alpha,n}(m,q,\lambda)f_s(z)]'}{[D^{\alpha,n}(m,q,\lambda)f_s(z)]^2}\right| \end{array}\right] + (1-|z|^2)[|\frac{1}{\gamma_1}|+\ldots+|\frac{1}{\gamma_s}|\ ]$$

$$= (1-|z|^2)\left[|\frac{1}{\gamma_1}|\left|\frac{z^2[D^{\alpha,n}(m,q,\lambda)f_1(z)]'}{[D^{\alpha,n}(m,q,\lambda)f_1(z)]^2}\right| - |\frac{1}{\gamma_1}| + |\frac{1}{\gamma_1}|\right] + \ldots$$

$$+ (1-|z|^2)\left[|\frac{1}{\gamma_s}|\left|\frac{z^2[D^{\alpha,n}(m,q,\lambda)f_s(z)]'}{[D^{\alpha,n}(m,q,\lambda)f_s(z)]^2}\right| - |\frac{1}{\gamma_s}| + |\frac{1}{\gamma_s}|\right] +$$

$$(1-|z|^2)[|\frac{1}{\gamma_1}|+\ldots+|\frac{1}{\gamma_s}|]$$

$$\le (1-|z|^2)\left[|\frac{1}{\gamma_1}|\left|\frac{z^2[D^{\alpha,n}(m,q,\lambda)f_1(z)]'}{[D^{\alpha,n}(m,q,\lambda)f_1(z)]^2} - 1\right|\right] + \ldots +$$

$$(1-|z|^2)\left[|\frac{1}{\gamma_s}|\left|\frac{z^2[D^{\alpha,n}(m,q,\lambda)f_s(z)]'}{[D^{\alpha,n}(m,q,\lambda)f_s(z)]^2} - 1\right|\right] +$$

$$(1-|z|^2)[|\frac{1}{\gamma_1}|+\ldots+|\frac{1}{\gamma_s}|] + (1-|z|^2)[|\frac{1}{\gamma_1}|+\ldots+|\frac{1}{\gamma_s}|]$$

$$\le (1-|z|^2)[|\frac{1}{\gamma_1}|+\ldots+|\frac{1}{\gamma_s}|] + 2(1-|z|^2)[|\frac{1}{\gamma_1}|+\ldots+|\frac{1}{\gamma_s}|]$$

$$\le 3(1-|z|^2)[|\frac{1}{\gamma_1}|+\ldots+|\frac{1}{\gamma_s}|]. \qquad (8)$$

From 8 and condition(i) ,we have

$$(1-|z|^2)\left|\frac{z[F^{\alpha,n}(m,q,\lambda,\gamma_i;z)]''}{[F^{\alpha,n}(m,q,\lambda,\gamma_i;z)]'}\right| \le 1, \text{ for all } z \in U.$$

By Lemma 1.2, it follows that the integral operator $F^{\alpha,n}(m,q,\lambda,\gamma_i;z)$ is univalent .

Setting $m=0$, $\alpha=0$, $n=0$, $\gamma_i=\frac{1}{\alpha_i}$ in Theorem 2.2, we obtain the following consequence of Theorem

Corollary 2.2 *For* $f_i \in A$, $i=\{1,2,3,\ldots,s\}$, *If* $f_i \in A$ *satisfy*

$$(i)\ |\alpha_1|+|\alpha_2|+\ldots+|\alpha_s|\leq\frac{1}{3},$$

$$(ii)\qquad |f_i(z)|\leq 1,$$

and

$$(iii)\ \left|\frac{z^2[f_i(z)]'}{[f_i(z)]^2}-1\right|<1.$$

For all $z\in U$, then the integral operator given by by Breaz and Breaz [9]. is univalent.

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